## MATHEMATICS

MD01
Unit Decision 1

Thursday 7 June 20079.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 3 and 5 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

## Instructions

- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The Examining Body for this paper is AQA. The Paper Reference is MD01.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.
- The final answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.


## Information

- The maximum mark for this paper is 75 .
- The marks for questions are shown in brackets.

Answer all questions.

1 Six people, $A, B, C, D, E$ and $F$, are to be matched to six tasks, $1,2,3,4,5$ and 6 . The following adjacency matrix shows the possible matching of people to tasks.

|  | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{A}$ | 0 | 1 | 0 | 1 | 0 | 0 |
| $\boldsymbol{B}$ | 1 | 0 | 1 | 0 | 1 | 0 |
| $\boldsymbol{C}$ | 0 | 0 | 1 | 0 | 1 | 1 |
| $\boldsymbol{D}$ | 0 | 0 | 0 | 1 | 0 | 0 |
| $\boldsymbol{E}$ | 0 | 1 | 0 | 0 | 0 | 1 |
| $\boldsymbol{F}$ | 0 | 0 | 0 | 1 | 1 | 0 |

(a) Show this information on a bipartite graph.
(b) At first $F$ insists on being matched to task 4. Explain why, in this case, a complete matching is impossible.
(c) To find a complete matching $F$ agrees to be assigned to either task 4 or task 5 .

Initially $B$ is matched to task $3, C$ to task $6, E$ to task 2 and $F$ to task 4 .
From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching.

2 (a) Use a Shell sort to rearrange the following numbers into ascending order, showing the new arrangement after each pass.

$$
\begin{array}{lllllllll}
28 & 22 & 20 & 17 & 14 & 11 & 6 & 5 & \text { (5 marks) }
\end{array}
$$

(b) (i) Write down the number of comparisons on the first pass.
(ii) Write down the number of swaps on the first pass.
(c) Find the total number of comparisons needed to rearrange the original list of 8 numbers into ascending order using a shuttle sort.
(You do not need to perform a shuttle sort.)

3 [Figure 1, printed on the insert, is provided for use in this question.]
The following network represents the footpaths connecting 12 buildings on a university campus. The number on each edge represents the time taken, in minutes, to walk along a footpath.

(a) (i) Use Dijkstra's algorithm on Figure 1 to find the minimum time to walk from $A$ to $L$.
(ii) State the corresponding route.
(b) A new footpath is to be constructed. There are two possibilities:
from $A$ to $D$, with a walking time of 30 minutes; or from $A$ to $I$, with a walking time of 20 minutes.

Determine which of the two alternative new footpaths would reduce the walking time from $A$ to $L$ by the greater amount.

4 The diagram shows the various ski-runs at a ski resort. There is a shop at $S$. The manager of the ski resort intends to install a floodlighting system by placing a floodlight at each of the 12 points $A, B, \ldots, L$ and at the shop at $S$.

The number on each edge represents the distance, in metres, between two points.


Total of all edges $=1135$
(a) The manager wishes to use the minimum amount of cabling, which must be laid along the ski-runs, to connect the 12 points $A, B, \ldots, L$ and the shop at $S$.
(i) Starting from the shop, and showing your working at each stage, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points.
(ii) State the length of your minimum spanning tree.
(1 mark)
(iii) Draw your minimum spanning tree.
(iv) The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and the eighth edges that the manager added to his spanning tree.
(b) At the end of each day a snow plough has to drive at least once along each edge shown in the diagram in preparation for the following day's skiing. The snow plough must start and finish at the point $L$.

Use the Chinese Postman algorithm to find the minimum distance that the snow plough must travel.
(6 marks)

5 [Figure 2, printed on the insert, is provided for use in this question.]
The Jolly Company sells two types of party pack: excellent and luxury.
Each excellent pack has five balloons and each luxury pack has ten balloons.
Each excellent pack has 32 sweets and each luxury pack has 8 sweets.
The company has 1500 balloons and 4000 sweets available.
The company sells at least 50 of each type of pack and at least 140 packs in total.
The company sells $x$ excellent packs and $y$ luxury packs.
(a) Show that the above information can be modelled by the following inequalities.

$$
x+2 y \leqslant 300, \quad 4 x+y \leqslant 500, \quad x \geqslant 50, \quad y \geqslant 50, \quad x+y \geqslant 140 \quad \text { (4 marks) }
$$

(b) The company sells each excellent pack for 80 p and each luxury pack for $£ 1.20$. The company needs to find its minimum and maximum total income.
(i) On Figure 2, draw a suitable diagram to enable this linear programming problem to be solved graphically, indicating the feasible region and an objective line.
(8 marks)
(ii) Find the company's maximum total income and state the corresponding number of each type of pack that needs to be sold.
(2 marks)
(iii) Find the company's minimum total income and state the corresponding number of each type of pack that needs to be sold.
(2 marks)

## Turn over for the next question

6 (a) Mark is staying at the Grand Hotel ( $G$ ) in Oslo. He is going to visit four famous places in Oslo: Aker Brygge ( $A$ ), the National Theatre $(N)$, Parliament House $(P)$ and the Royal Palace ( $R$ ).

The figures in the table represent the walking times, in seconds, between the places.

|  | Grand <br> Hotel ( $\boldsymbol{G})$ | Aker <br> Brygge $(\boldsymbol{A})$ | National <br> Theatre $(\boldsymbol{N})$ | Parliament <br> House (P) | Royal <br> Palace (R) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Grand <br> Hotel ( $\boldsymbol{G})$ | - | 165 | 185 | 65 | 160 |
| Aker <br> Brygge (A) | 165 | - | 155 | 115 | 275 |
| National <br> Theatre (N) | 185 | 155 | - | 205 | 125 |
| Parliament <br> House (P) | 65 | 115 | 205 | - | 225 |
| Royal <br> Palace (R) | 160 | 275 | 125 | 225 | - |

Mark is to start his tour from the Grand Hotel, visiting each place once before returning to the Grand Hotel. Mark wishes to keep his walking time to a minimum.
(i) Use the nearest neighbour algorithm, starting from the Grand Hotel, to find an upper bound for the walking time for Mark's tour.
(ii) By deleting the Grand Hotel, find a lower bound for the walking time for Mark's tour.
(iii) The walking time for an optimal tour is $T$ seconds. Use your answers to parts (a)(i) and (a)(ii) to write down a conclusion about $T$.
(b) Mark then intends to start from the Grand Hotel $(G)$, visit three museums, Ibsen ( $I$ ), Munch $(M)$ and Viking $(V)$, and return to the Grand Hotel. He uses public transport. The table shows the minimum travelling times, in minutes, between the places.

| From | Grand Hotel <br> $(\boldsymbol{G})$ | Ibsen <br> $(\boldsymbol{I})$ | Munch <br> $(\boldsymbol{M})$ | Viking <br> $(\boldsymbol{V})$ |
| :---: | :---: | :---: | :---: | :---: |
| Grand Hotel $(\boldsymbol{G})$ | - | 20 | 17 | 30 |
| Ibsen $(\boldsymbol{I})$ | 15 | - | 32 | 16 |
| Munch $(\boldsymbol{M})$ | 26 | 18 | - | 21 |
| Viking $(\boldsymbol{V})$ | 19 | 27 | 24 | - |

(i) Find the length of the tour GIMVG.
(ii) Find the length of the tour $G V M I G$.
(iii) Find the number of different possible tours for Mark.
(iv) Write down the number of different possible tours for Mark if he were to visit $n$ museums, starting and finishing at the Grand Hotel.

## END OF QUESTIONS

There are no questions printed on this page


General Certificate of Education June 2007
Advanced Subsidiary Examination

MATHEMATICS
MD01
Unit Decision 1

## Insert

Insert for use in Questions 3 and 5.
Fill in the boxes at the top of this page.
Fasten this insert securely to your answer book.

Figure 1 (for use in Question 3)


Figure 2 (for use in Question 5)


There is no text printed on this page

